Int. J. Heat Mass Transfer. Vol. 35, No. 9, pp. 2145–2159, 1992
Printed in Great Britain

0017–9310/92 $5.00+0.00
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Wavenumber selection for Rayleigh–Bénard convection in a small aspect ratio box

D. MUKUTMONI† and K. T. YANG

Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, IN 46556, U.S.A.

(*Received* 19 *July* 1991 *and in final form* 29 *August* 1991)

Abstract—The present numerical study is directed towards Rayleigh–Bénard convection with insulated side walls for small aspect ratio enclosures. A Boussinesq fluid is assumed. Bifurcations have been documented and the physics of the flow field have been looked into for three different aspect ratios, 3.3 : 1.9 : 1, 3.5 : 2.1 : 1 and 4 : 2 : 1, and three different fluid Prandtl numbers, 0.5, 0.71 and 2.5. It was found that rolls parallel to the long side are stable only below a Rayleigh number of 20 000 for a 3.5 : 2.1 : 1 geometry with water ($Pr = 2.5$) as the fluid. For the 3.3 : 1.9 : 1 geometry with liquid helium ($Pr = 0.5$) the transition from a three-cell to a two-cell pattern is accompanied by strong time dependence. For the 4 : 2 : 1 geometry with air ($Pr = 0.71$) the transition from four to three rolls was found to be a result of the skewed varicose instability.

INTRODUCTION

THE IMPORTANCE of Rayleigh–Bénard convection, i.e. natural convection in the heated from below case, is well recognized in the literature. Aside from its relevance to some practical applications, it is inherently unstable and provides a unique and useful physical situation to study the dynamics of a non-linear system governed by partial differential equations. One striking non-linear phenomenon of Rayleigh–Bénard convection is the so-called pattern selection process [1]. This refers to the observation that even for the same set of parameters the system generates a rich selection of different flow structures that are breathtaking in their variety.

One typical flow pattern which has received considerable attention in the past is roll convection [2]. As shown in Fig. 1, the flow consists of two-dimensional cells (with the exception of wall effects) that are roughly square in cross-section. In this paper we look into some of the critical issues of roll convection with the help of direct numerical simulations and, in particular, roll convection and pattern selection in small aspect ratio enclosures (aspect ratios less than 5). We basically address two related issues in the pattern selection problem of rolls, otherwise known as the wavenumber selection problem.

Firstly, early theoretical [3] and experimental results [4] seem to suggest that convective rolls in rectangular boxes invariably align themselves parallel to the shorter horizontal dimension of the rectangular enclosure. Later theoretical results [5] showed in a definitive manner that although rolls parallel to the short side are more likely (henceforth referred to as short rolls), rolls parallel to the long side do exist (henceforth referred to as long rolls). Furthermore, Kolodner *et al.* [6] did in fact observe long rolls experimentally for intermediate and large Prandtl number fluids. This paper looks into the stability of long rolls for moderate Prandtl number fluids by direct numerical simulation.

This is an important issue. Small aspect ratio RB convection with short rolls have been used in the past [7, 8] to study the various routes to chaos and turbulence. If stable long roll patterns can indeed be generated this could be a useful starting point for further investigations in the time-dependent domain. Assuming that the flow structure retains its stability, further insights into the transition phenomenon can be found by perhaps uncovering some additional routes to turbulence. On the other hand, if stable and regular flow patterns cannot be maintained, it is of interest to know the mechanism as well as the outcome of the change as the Rayleigh number is increased.

The second related question is with respect to the curious phenomenon of 'loss of rolls' observed experimentally for intermediate and large aspect ratio boxes

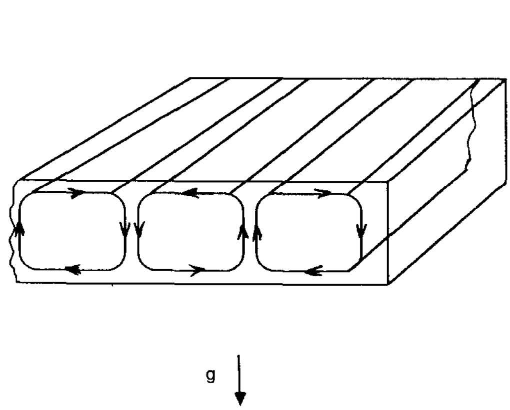

FIG. 1. Schematic diagram of Rayleigh–Bénard convection in the form of rolls.

† Current address: Department of Mechanical Engineering, Naval Postgraduate School, Monterey, CA 93943, U.S.A.

NOMENCLATURE

A_x	aspect ratio in the x-direction	v	non-dimensional y-direction velocity
A_z	aspect ratio in the z-direction	w	non-dimensional z-direction velocity
g	acceleration due to gravity [$m^2 s^{-1}$]	x	non-dimensional horizontal spatial coordinate
L	height of the enclosure [m]	y	non-dimensional vertical spatial coordinate
Nu	Nusselt number	z	non-dimensional spatial coordinate in the direction of depth.
p	non-dimensional pressure		
Pr	ν/α, Prandtl number		
Ra	$g\beta\Delta TL^3/\nu\alpha$, Rayleigh number		
t	non-dimensional time		
T	non-dimensional temperature	Greek symbols	
T_C	cold wall temperature [°C]	α	thermal diffusivity [$m^2 s^{-1}$]
T_H	hot wall temperature [°C]	β	coefficient of volume expansion [K^{-1}]
T_m	$(T_H+T_C)/2$, mean temperature [°C]	ΔT	temperature difference, T_H-T_C [°C]
$\mathbf{U}$	non-dimensional velocity vector	ν	kinematic viscosity [$m^2 s^{-1}$]
u	non-dimensional x-direction velocity	ρ	density [$kg\ m^{-3}$].

by many [6, 9, 10]. This relates to the fact that the wavenumber of the rolls decreases with an increase in Rayleigh number as a consequence of discrete transitions (decrease) in the number of rolls. Busse and Clever [11] showed theoretically that this is a consequence of the skewed varicose instability for RB convection without side walls. Nevertheless, the phenomenon is still not well understood for RB convection with side walls. Some doubts still remain, particularly since some of the earlier analyses [12] predicted just the opposite, i.e. an increase of wavenumber with an increase in Rayleigh number.

In this paper, this phenomenon has also been looked into with the aid of numerical simulation but is limited to small boxes. We are not aware of any previous attempts at numerical simulations of the roll transition phenomenon, although there have been failures [13]. A numerical investigation can potentially provide much more insight into the problem since it is possible to take a look at the flow and temperature field to an extent that cannot be matched experimentally. A numerical investigation was therefore undertaken keeping these things in mind.

GOVERNING EQUATIONS AND FORMULATIONS

The geometry of the enclosure is shown in Fig. 1. The vertical walls are all adiabatic. The bottom wall is heated and the top wall is cooled, both isothermally. The fluid is Boussinesq, i.e. we assume that all transport properties of the fluid are constant with the exception of the buoyancy term in the momentum equations, which is linearized. The governing equations are non-dimensionalized by suitable scales of the dependent and independent variables. The x, y, and z coordinates were scaled by L, the enclosure height, the velocities were scaled by α/L, the time by L^2/α, and the equilibrium hydrostatic pressure in the absence of a temperature gradient was scaled by $\rho\alpha^2/L^2$. The temperature was non-dimensionalized by $T-T_m/\Delta T$, $\Delta T = T_H - T_C$, and $T_m = (T_H+T_C)/2$, α is the thermal diffusivity and ρ is the fluid density. The non-dimensionalized governing equations for the Boussinesq equations are the following [2]:

$$\nabla\cdot\mathbf{U} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\nabla\cdot(u\mathbf{U}) = -\frac{\partial p}{\partial x}+Pr\,\nabla^2 u \tag{2}$$

$$\frac{\partial v}{\partial t}+\nabla\cdot(v\mathbf{U}) = -\frac{\partial p}{\partial y}+Pr\,\nabla^2 v+Ra\,Pr\,T \tag{3}$$

$$\frac{\partial w}{\partial t}+\nabla\cdot(w\mathbf{U}) = -\frac{\partial p}{\partial z}+Pr\,\nabla^2 w \tag{4}$$

$$\frac{\partial T}{\partial t}+\nabla\cdot(T\mathbf{U}) = \nabla^2 T. \tag{5}$$

The boundary conditions consistent with the adiabatic and isothermal walls in a non-dimensional form are the following:

$$x = 0, A_x;\quad 0 \leqslant z \leqslant A_z,\quad 0 \leqslant y \leqslant 1$$

$$u = v = w = 0\quad \frac{\partial T}{\partial x} = 0 \tag{6}$$

$$z = 0, A_z;\quad 0 \leqslant x \leqslant A_x;\quad 0 \leqslant y \leqslant 1$$

$$u = v = w = 0\quad \frac{\partial T}{\partial z} = 0 \tag{7}$$

$$y = 0, 1;\quad 0 \leqslant x \leqslant A_x;\quad 0 \leqslant z \leqslant A_z;$$

$$u = v = w = 0\quad T = 0.5-y. \tag{8}$$

Note that the non-dimensional pressure is actually the pressure difference between the local and the hydrostatic pressure under isothermal and quiescent conditions.

The governing equations are solved in primitive variables in a uniform, three-dimensional staggered grid based on the control volume method [14]. The QUICK scheme is used in the finite difference formulation of the convective terms to minimize numerical diffusion effects [15]. The SIMPLEX algorithm [16] was used to solve the coupled heat transfer and fluid flow problem which is essentially a more implicit variant of SIMPLE. The time step was typically 0.001.

VALIDATION AND GRID REFINEMENT

For three-dimensional RB convection in a box there is no analytical solution for even the limiting cases. It therefore becomes necessary to compare the numerical solution with experiments. The interferometric results of Farhadieh and Tankin [17] were used for comparison. The details of the validation are given elsewhere [18]. Internal checks on the accuracy and consistencies show that the average Nusselt numbers at the cold and hot walls, which is a test of global energy balance, were equal to within machine precision when steady state was reached. The continuity equation was satisfied for every control volume to within machine precision as well. A detailed grid refinement study has been reported elsewhere [19] and will not be repeated here. The study revealed that a horizontal resolution of 0.1 and a vertical resolution of 0.05 were a good compromise between accuracy and computational expenses.

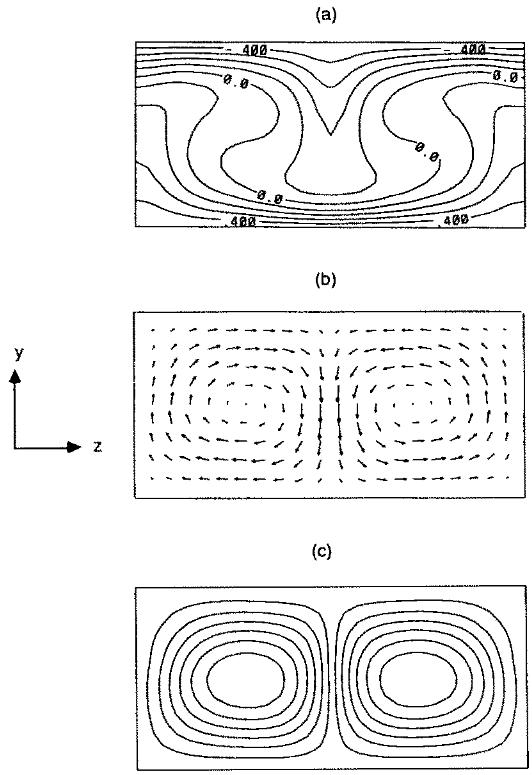

FIG. 2. Isotherms (a), velocity vectors (b) and sectional streamlines (c) for $Ra = 14\,000$ at the vertical plane $z = 1.44$ for long rolls.

STABILITY OF LONG ROLLS

We study the stability of long rolls by numerical simulation. The aspect ratios are taken to be 3.5 and 2.1 and the Prandtl number is set at 2.5. These parameters are the same as in the experiments of Gollub and Benson [7]. However, in their investigation the flow field consisted of two counter-rotating rolls parallel to the short side. In contrast what we have here are two counter-rotating rolls parallel to the long side. The two are the same except that the wavenumber of the long rolls is somewhat larger. The two-roll structure is generated with velocity perturbations (Fig. 2) as given in Mukutmoni and Yang [18]. All computations were carried out with a $30 \times 20 \times 30$ grid, which is consistent with the grid refinement study.

In the two-roll structure, the fluid rises along the side walls and descends along the plane of symmetry in between the rolls. The flow is mostly two-dimensional except near the walls and some weakly axial flow as mentioned in Mukutmoni and Yang [19]. The numerical investigation was carried out as follows: computations were started with a Rayleigh number of 4000. Then, using the steady state velocity and temperature field as an initial condition, the Rayleigh number was increased in steps of 2000. The other parameters, which are the aspect ratios and Prandtl number, were of course not changed.

It was found that between Rayleigh numbers of 18 000 and 20 000 the two-roll structure changed dramatically. The outcome of the change is a structure that is fully three-dimensional (Fig. 3) as a result of secondary flows parallel to the roll axis. The planform structure can be described as four rectangular cells each having the same aspect ratio as that of the enclosure itself. Thus, unlike the rolls, the spatial periodicity is present in both directions [20].

The computed pathlines shown in Fig. 4 give some additional information about the flow topology. It appears that the flow is rising along all four side walls and descending at the center of the horizontal planform. A perfect four-fold symmetry that was evident before the transition is still preserved and the flow field is still time independent. As the Rayleigh number is increased further the flow field becomes oscillatory and also the mean flow suffers qualitative structural changes between Rayleigh numbers of 34 000 and 36 000.

As seen in Fig. 5, the mean flow can now be described as two rectangular cells in the horizontal planform. Thus, it now has a two-fold symmetry about one of the vertical planes. Symmetry breaking bifurcation is typical of RB convection [2]. The Nusselt numbers as a function of the Rayleigh number are given in Fig. 6. Note that we see no discontinuity in the Nusselt number as a result of the transition from the roll planform to a rectangular planform. The flow

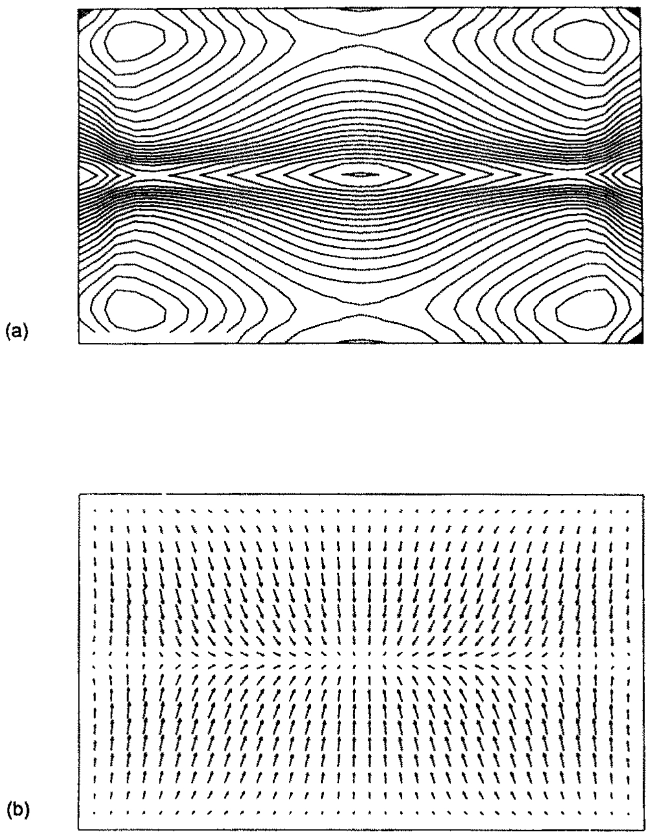

FIG. 3. Isotherms (a) and velocity vectors (b) at the horizontal section $y = 0.8$ for $Ra = 24\,000$.

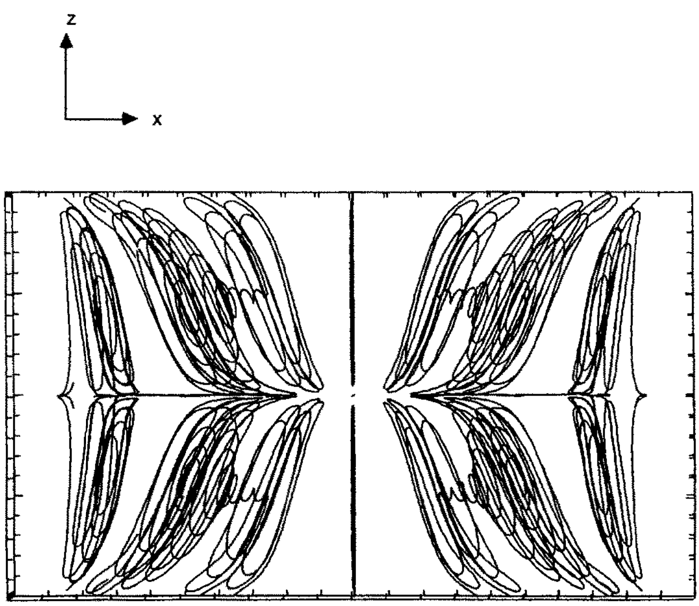

FIG. 4. Top perspective view of computed pathlines for $Ra = 24\,000$.

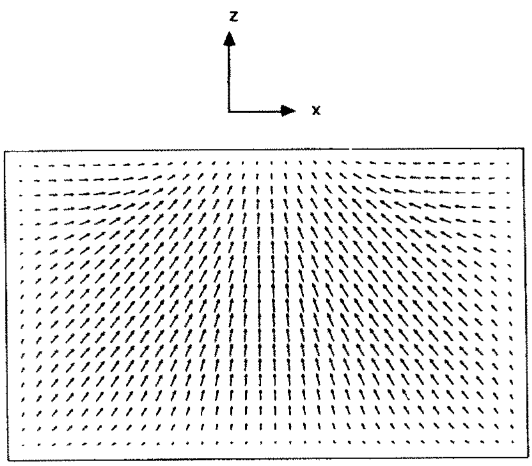

FIG. 5. The mean velocity field for $Ra = 36\,000$ at the horizontal section $y = 0.8$.

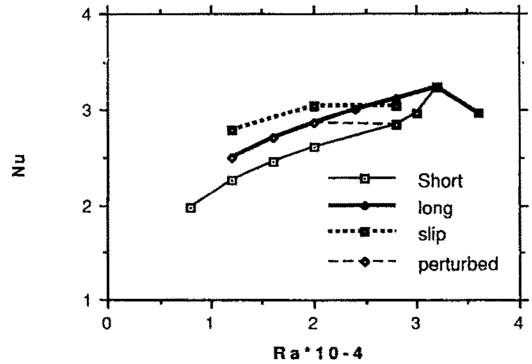

FIG. 6. The average Nusselt number as a function of the Rayleigh number for long rolls. (a) Rigid vertical walls without forcing. (b) Rigid vertical walls with forcing. (c) Vertical slip walls with forcing. (d) Rigid vertical walls for short rolls.

transition from an essentially two-dimensional flow structure to a three-dimensional one is a novel phenomenon and has not been documented for small aspect ratio boxes and moderate Prandtl numbers, either experimentally or numerically. However, the phenomenon is not surprising. Kolodner *et al.* [6] documented a transition from long rolls to bimodal convection for a high Prandtl number fluid, which is similar to the transition that one observes in this case.

Transition from long rolls to short rolls reported in Mukutmoni and Yang [18] can be simulated between Rayleigh numbers of 24 000 and 26 000. However, finite perturbations were required. The perturbations generated from the calculations so far caused by machine round-off errors are practically infinitesimal. For finite perturbations, the Rayleigh number of the fluid was increased abruptly to a very high value and then brought back to the same state. Physically, this means that the fluid has been suddenly heated and then cooled. This 'quenching' operation has been used by Giglio *et al.* [21] in their experiments. Note that if this approach is not used, the rectangular cell pattern was found to be quite stable and no transition occurred. Therefore, it can be said that the rectangular cell pattern is metastable, i.e. it is stable to infinitesimal perturbations but not finite ones.

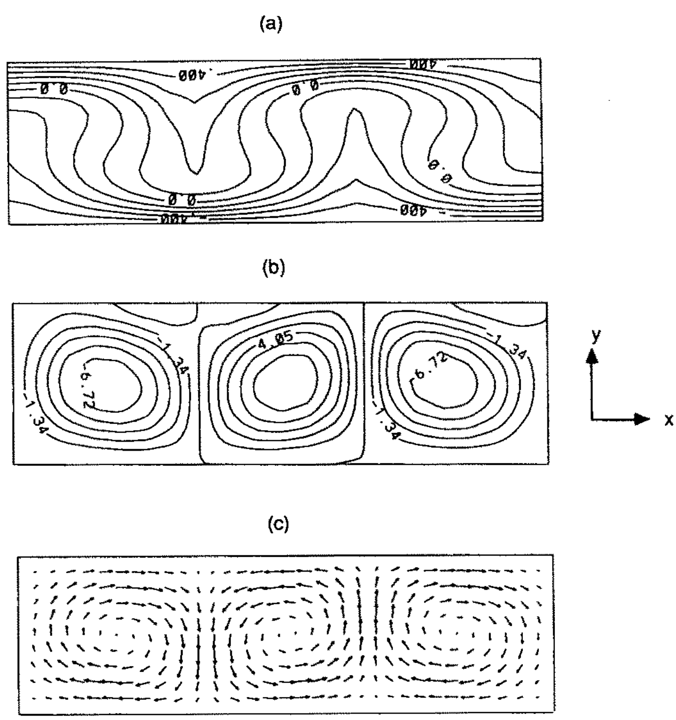

FIG. 7. Isotherms (a), sectional streamlines (b) and velocity vectors (c) for $Ra = 14\,000$ at the vertical plane $z = 1.6$.

The Nusselt number as a function of the Rayleigh number, which shows the forced transition from long to short rolls, is depicted in Fig. 6. Note that after transition there is a drop in the Nusselt number. This is to be expected, since the wavenumber after the transition has decreased. For the purpose of comparison, the Nusselt number variation for the short rolls is also shown. The flow field after transition is indistinguishable from a short roll pattern. If the vertical side walls are assumed to be stress-free, the same qualitative behavior is observed, except that the two transitions occur at lower Rayleigh numbers. Also, the Nusselt numbers for the same Rayleigh number are higher (Fig. 6).

It is known that the Prandtl number of the fluid significantly affects the transition phenomena. For that reason we now study the stability of roll convection for liquid helium, which has a much lower Prandtl number of 0.5. However, we restrict ourselves to short rolls for which experiments have been performed, most notably by Libchaber and co-workers.

TRANSITIONS IN LIQUID HELIUM

One of the relevant experiments is given in Maurer and Libchaber [8] for small aspect ratio enclosures. The geometry of the cell is a parallelepiped cell with a base 1.6×2.8 mm and a height of 0.85 mm and thus has a $3.3:1.9:1$ geometry. The flow field at the onset of convection is a three short roll structure. In the experiments, this was dislodged by increasing the Rayleigh number to around 4×10^4 and then reducing it back to the earlier value (quenching operation). This resulted in a two-roll structure that was required for their experiments. The question that we ask here is the following: can the transition from three rolls to two rolls be accomplished without the rapid heating and cooling sequence? Do we observe a three-dimensional pattern after transition? The Prandtl number of liquid helium is 0.5 for a temperature of 3 K and a pressure of three atmospheres as in the experiments.

To that end, a three-roll structure was generated by perturbations at a Rayleigh number of 6000 and there-

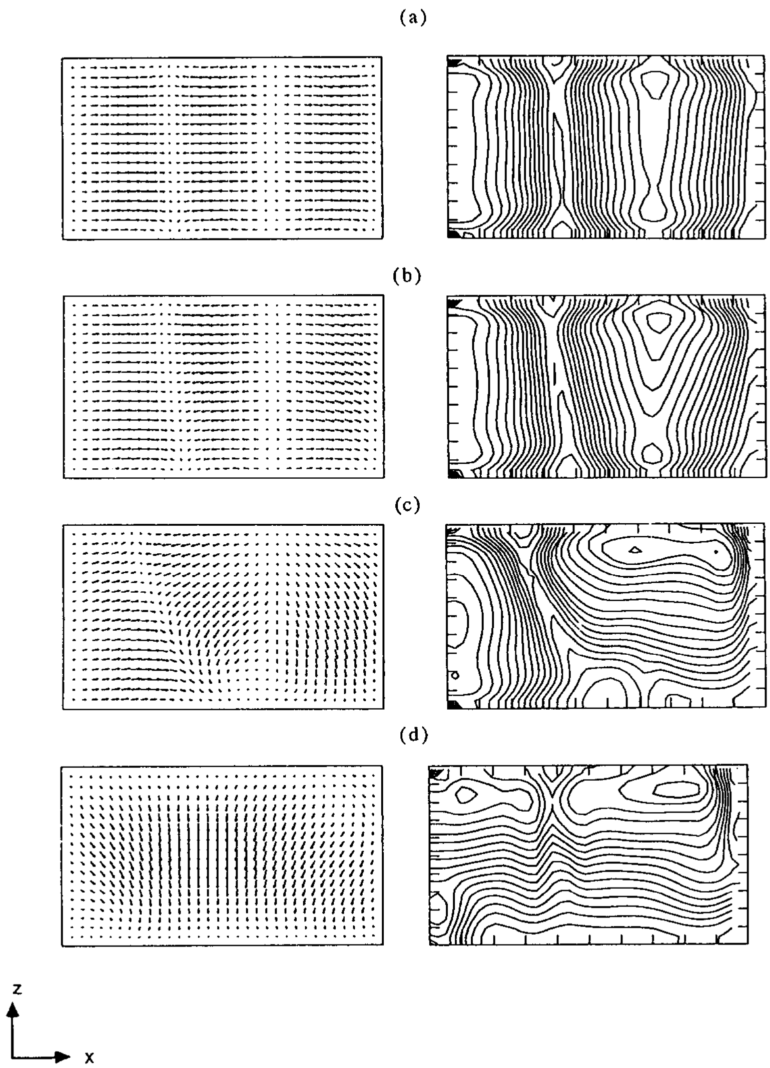

FIG. 8. Transition sequence from three to two rolls. Time interval of 0.6, $Ra = 16\,000$.

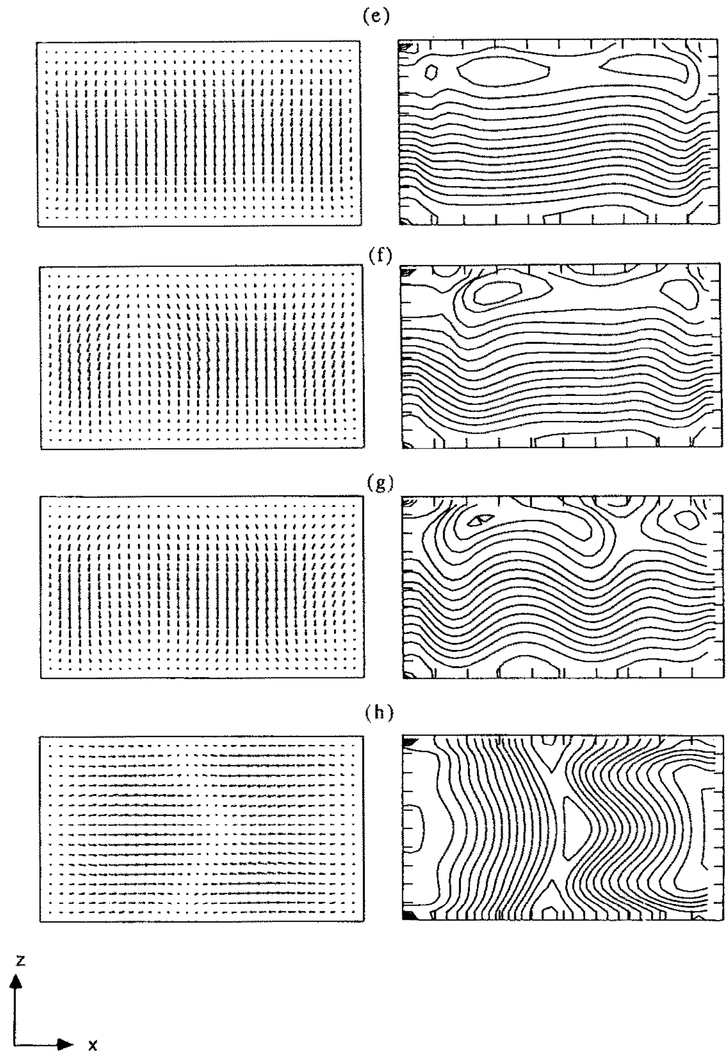

FIG. 8.—*Continued.*

after increased in steps of 2000. A $30 \times 20 \times 20$ grid for a 3.3 : 1.9 : 1 box of the experiments was used for the computations. The three-cell pattern shown in Fig. 7 remains unchanged up to a Rayleigh number of 14 000. The three-roll convective pattern was found to unravel at a Rayleigh number of 16 000. The instability sequence is shown in Fig. 8. The sequence shown is at time intervals of 0.6 except for the last set. The first three sequences seem to indicate that the SV instability is in place. We observe the typical slanting of the roll axis. The next four sequences are, however, more complicated. It seems like the flow pattern was getting towards a single cell pattern parallel to the long side. However, long time simulation clearly indicates that the asymptotic flow pattern is actually an oscillating two-cell structure parallel to the short side. An instantaneous snapshot of the oscillating flow is shown in Fig. 8(h). Thus, a three-dimensional pattern is not observed and the transition is from three to two rolls. The mean two-roll flow pattern after the transition is shown in Fig. 9.

The oscillating flow and temperature field can now be analyzed in greater detail. The velocity field at a vertical section $z = 1.6$ for one complete time period is shown in Fig. 10. The time period is roughly 0.17. The oscillating isotherms for one complete period are shown for the horizontal section $y = 0.75$ in Fig. 11. The results show clearly that the oscillations are in the form of stationary waves parallel to the roll axis and similar to that of a vibrating string. The side walls are the nodes and the core region where the oscillation is maximum is called the antinode. The rolls are mostly oscillating up and down. A considerable amount of roll excursion was also seen by Upson *et al.* [13] in their computations.

Note that flow visualization is not possible for liquid helium. Based on the work of others, as mentioned by Maurer and Libchaber [8], it was concluded

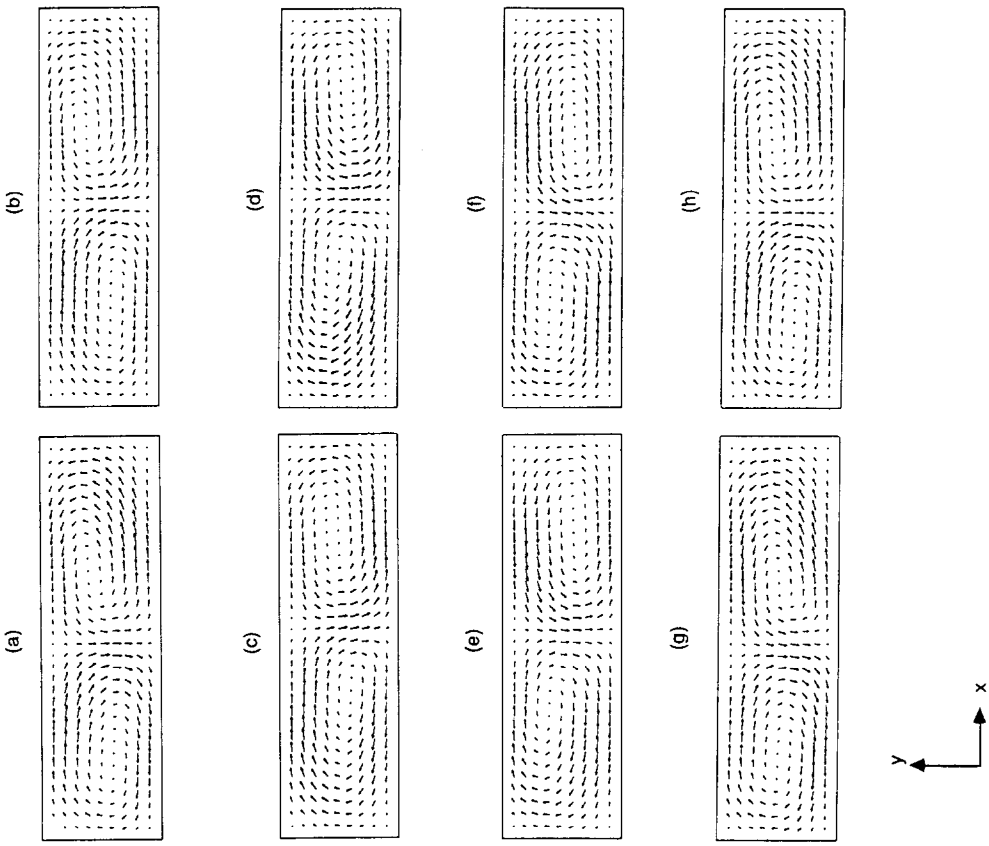

FIG. 10. The oscillating flow field over one complete period shown for the vertical section $z = 1.6$. The time interval is 0.023.

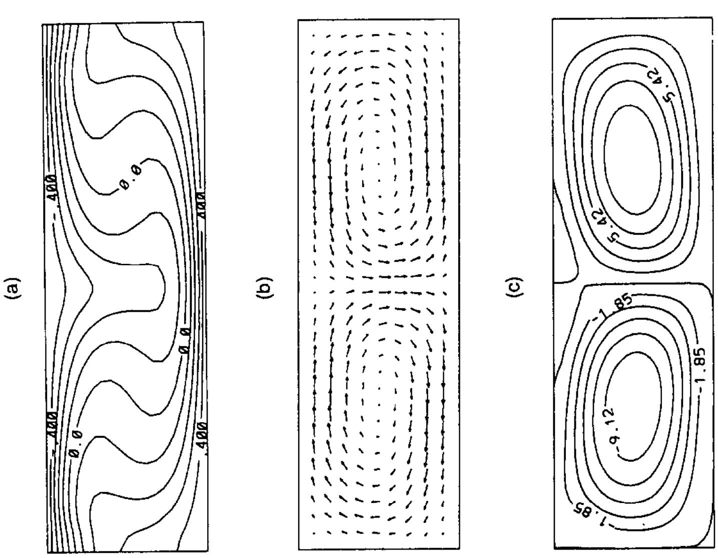

FIG. 9. Isotherms (a), velocity vectors (b) and sectional streamlines (c) for $Ra = 18\,000$ (mean flow) at the vertical plane $z = 1.44$.

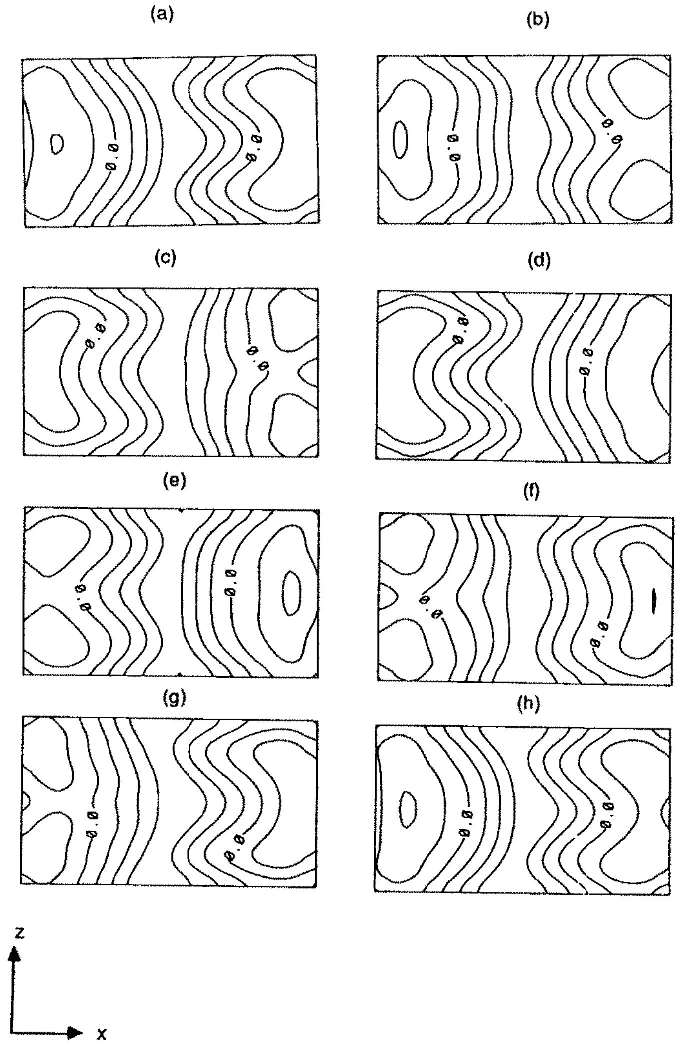

FIG. 11. The oscillating isotherms over one complete period at the horizontal section $y = 0.8$. Time interval is 0.023.

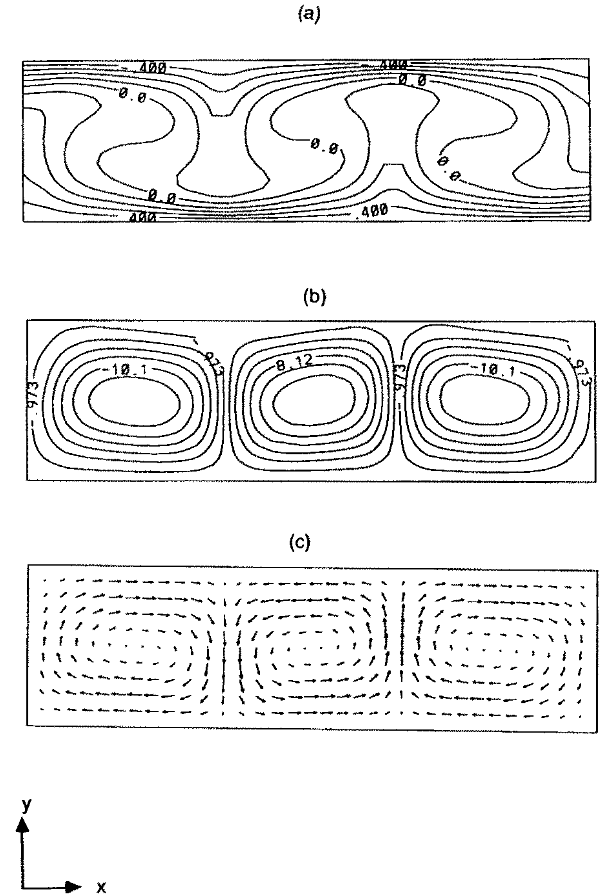

FIG. 12. Isotherms (a), sectional streamlines (b) and velocity vectors (c) for $Ra = 20\,000$ at the vertical plane $z = 1.5$ for water.

that the oscillation is in the form of travelling waves. Based on the results of this simulation we can now say with confidence that this is not the case. A standing wave pattern that is a superposition of travelling waves generated at the core region and subsequent reflection off the side walls perpendicular to the roll axis is what was observed.

THREE-ROLL TRANSITION FOR WATER

In this section the stability of a three-roll convection for the same parameters as used in the experiments of GB is examined. Consequently, the Prandtl number of the fluid is taken to be 2.5 for a 3.5 : 2.1 : 1 box. It is intriguing that a three-roll convection pattern for a Prandtl number of 5.0 was presented by GB, but no such pattern was reported for $Pr = 2.5$. Does that mean that a stable three-roll pattern does not exist for $Pr = 2.5$? In this section we examine the stability of the three-roll pattern by numerical simulations.

The three-roll convection shown in Fig. 12 was generated as usual by velocity perturbations. The purpose of the simulation was to see if there was a transition from a three-roll to a two-roll configuration as in the case of liquid helium. A $30 \times 20 \times 20$ grid was used for these sets of simulations.

The results demonstrate that up to a Rayleigh number of 24 000, the three-roll configuration is quite stable. The flow becomes unstable between Rayleigh numbers of 24 000 and 26 000. The sequence of instability is shown in Figs. 13(a)–(f), also in the form of velocity vectors and isotherms for the same location. The instability mechanism is the same SV pattern that was observed in the long roll transition sequence. The final result is a pattern consisting of three rectangular cells, as shown in Fig. 14. The rectangular cell pattern remained stable up to a Rayleigh number of 50 000. At that Rayleigh number the flow breaks down.

The break down is accompanied by strong time dependence. The runs were continued until the asymptotic state was reached. The results are shown in Figs. 15(a) and (b), which correspond to a grid location of (10,7,7). We observe broadband noise and the phase trajectory is highly irregular. A Lyapunov exponent calculation based on the algorithm given in Wolf *et al.* [22] shows that the largest Lyapunov exponent is positive. Although the flow is aperiodic, as can be seen in Fig. 15(a), the mean flow does show some structure. The flow structure can be described as cellular convection that has one big enclosure size rectangular cell. This bifurcation thus represents a change directly from steady to chaotic convection that is sometimes known as snap-through bifurcation [2]. This sequence is similar to what is observed in the case of intermediate and large aspect ratio boxes [23]. The effect of the fluid Prandtl number on the stability and subsequent evolution of a three-roll pattern is therefore quite dramatic.

FOUR-ROLL TRANSITION FOR AIR

The final transition sequence simulated draws from some of the experimental results due to Kirchartz and Oertel [9]. In that paper, among other things the loss of roll phenomena was examined experimentally. Experiments were carried out for three different fluids (air, water and silicone oil) and two different geometries (10 : 4 : 1 and 4 : 2 : 1). Of these, the transition from four rolls to three rolls in a 4 : 2 : 1 box for air ($Pr = 0.71$) has been chosen for simulation purposes. A $40 \times 20 \times 20$ grid was selected and this conforms to the prescription of the grid refinement study.

Experimentally, there is one major difference: the boundary conditions at the side wall are closer to those of infinite conductivity than adiabatic conditions. Therefore, the simulations do not exactly match the experimental conditions. The purpose of the simulations is not to duplicate the experimental results but rather to see what would happen if the side walls were insulated instead. Note that such a boundary condition is difficult to impose for air since the thermal conductivity of air is very small. Thus additional insights could be obtained and this would be one of the many ways in which simulations complement experiments.

In this study, a four-roll pattern (Fig. 16) was generated parallel to the short side by the usual technique of velocity perturbations. The simulations were started with a Rayleigh number of 4000, progressively increased in steps of 2000. The stability of the four-roll configuration was thus studied by numerical computations. The four-roll structure is stable up to a Rayleigh number of 8000. Between Rayleigh numbers of 8000 and 10 000 the flow loses stability.

That the flow changes as a result of this instability is shown in Fig. 17. The instability is unmistakably skewed varicose with the typical slanting of the rolls that ensues. What we therefore see is the same four-roll to three-roll transition but at a Rayleigh number which is much lower. It seems that having perfectly conducting walls does not really alter the physics of the transition process. Rather, it allows the four-roll pattern to retain its stability for higher Rayleigh numbers.

DISCUSSION AND CONCLUSIONS

From the results of the simulation, it can be said that the effect of the wall is to stabilize fully three-dimensional patterns in the case of moderate Prandtl number fluids such as water, whereas three-dimensional patterns were possible only for high Prandtl number fluids [6, 24]; friction at the walls makes it possible to have rectangular cell patterns for Prandtl number fluids that are lower. However, if the Prandtl number is sufficiently low, as in the case of liquid helium and air, no three-dimensional patterns can be observed in the time-averaged sense. The flow pattern was always found to be roll-like.

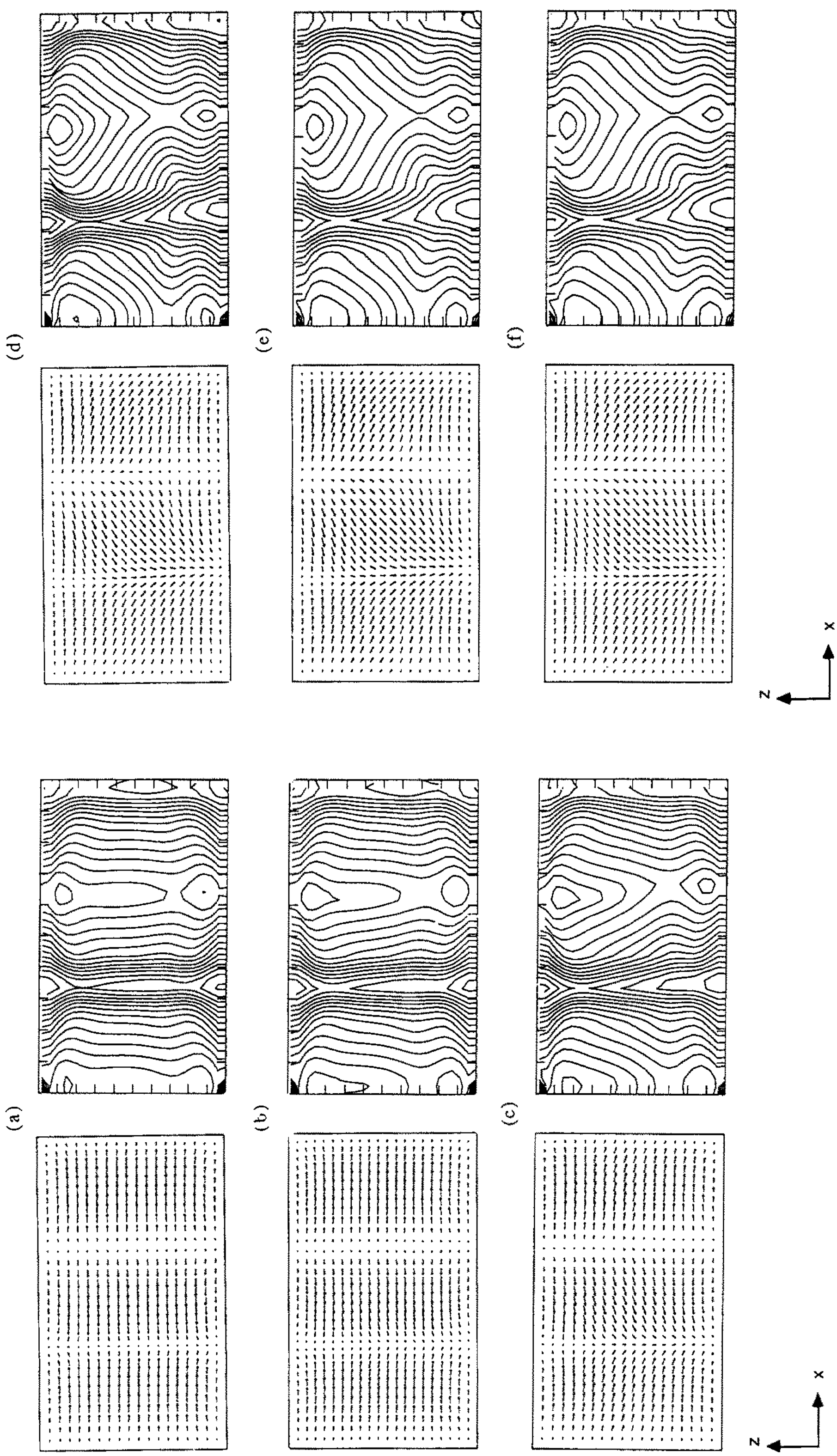

FIG. 13. The instability sequence at $Ra = 26\,000$.

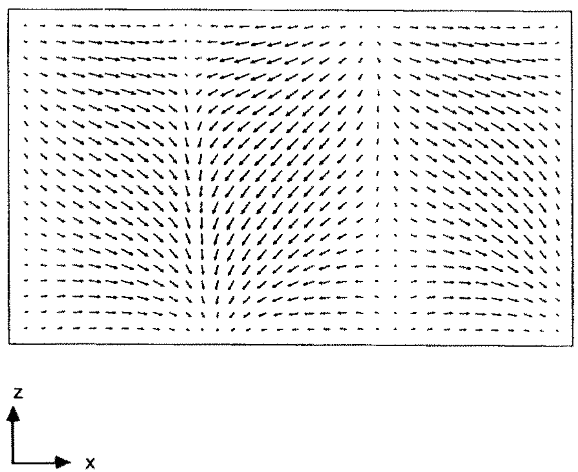

FIG. 14. Velocity field of the rectangular cell pattern at the horizontal section $y = 0.8$.

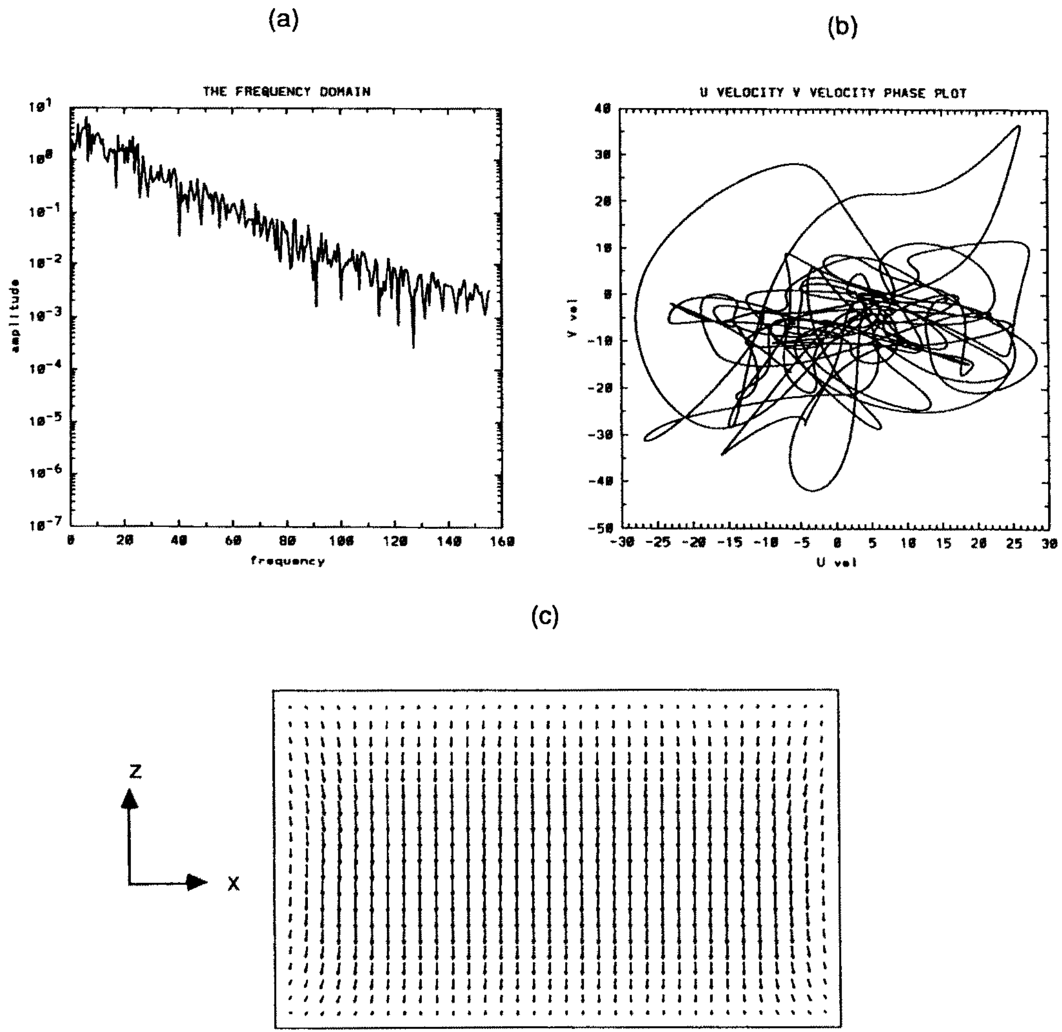

FIG. 15. (a) Frequency spectra. (b) Phase trajectory at the grid location (10,7,7). (c) Mean flow at the horizontal plane $y = 0.8$.

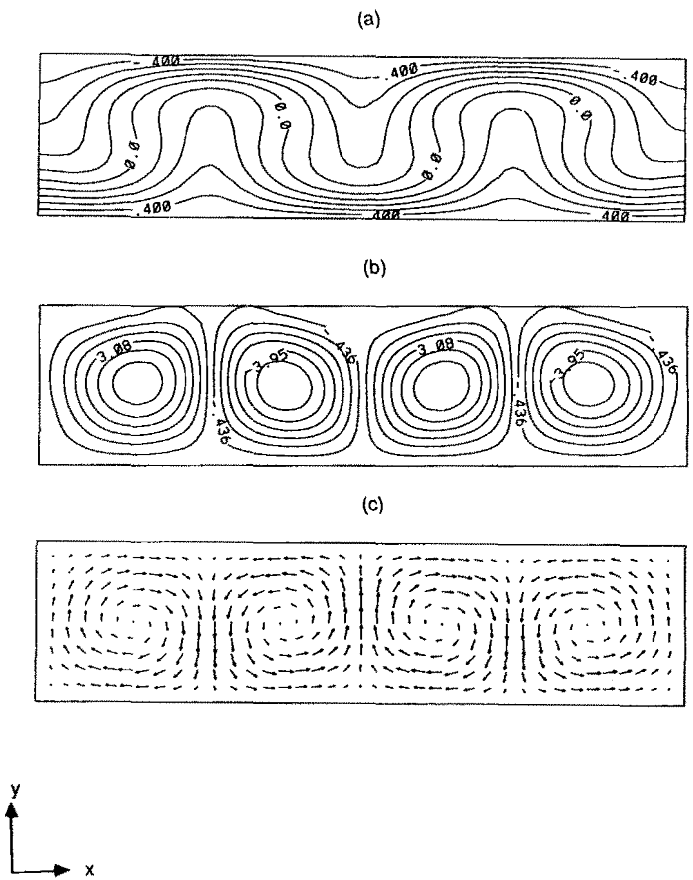

FIG. 16. Four-cell pattern for air. Isotherms (a), sectional streamlines (b) and velocity vectors (c) for $Ra = 6000$ at the vertical plane $z = 1.6$.

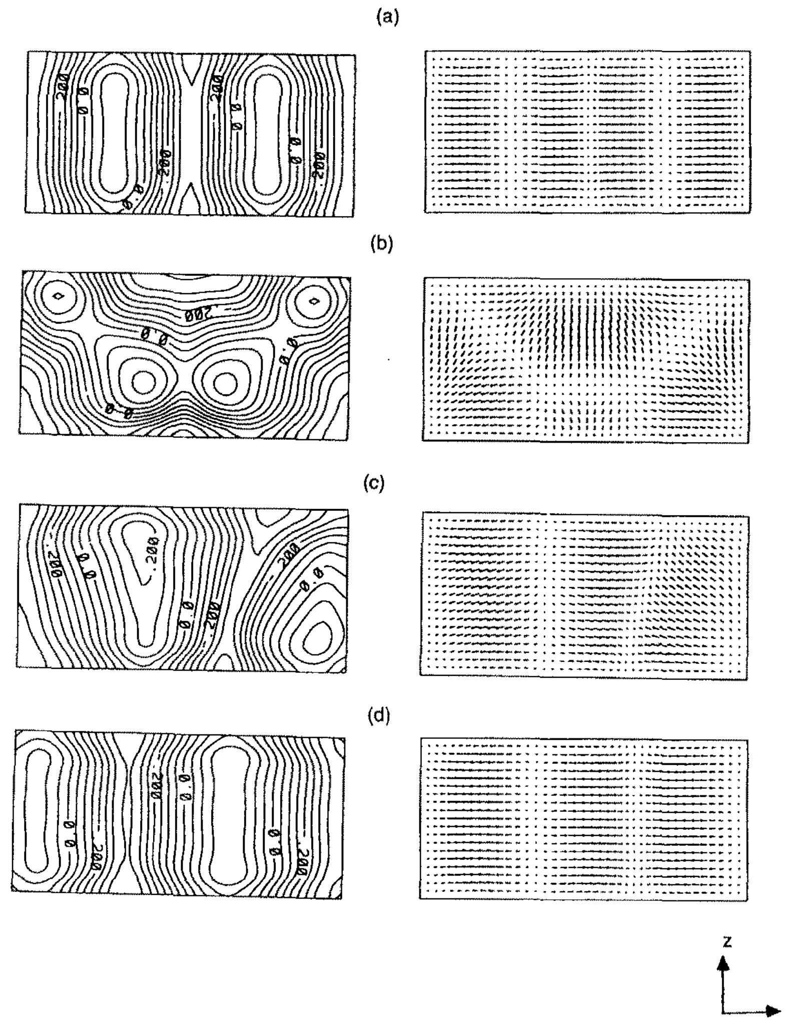

FIG. 17. Transition sequence from four to three cells. $Ra = 10\,000$; horizontal section $y = 0.8$.

With respect to long rolls it can be said that stable long rolls for small aspect ratio boxes are possible but only below a certain Rayleigh number. The flow eventually undergoes two separate bifurcations and becomes oscillatory. The dynamical behavior of the flow beyond the oscillatory regime is currently being looked into.

Acknowledgements—The computations were done on a CONVEX C240 supercomputer and IBM RS/6000 workstation. The support of the Office of University Computing of the University of Notre Dame is greatly appreciated. Some of the calculations were carried out on the Cray Y/MP at the National Center for Supercomputing Applications. Their support under grant ATM89007n is graciously acknowledged.

REFERENCES

1. I. Catton, Wavenumber selection in Bénard convection, *J. Heat Transfer* **110,** 1154–1965 (1988).
2. K. T. Yang, Transitions and bifurcations in laminar buoyant flows in confined enclosures, *J. Heat Transfer* **110,** 1191–1204 (1988).
3. S. H. Davis, Convection in a box: linear theory, *J. Fluid Mech.* **30,** 465–478 (1967).
4. K. Stork and U. Mueller, Convection in boxes: experiments, *J. Fluid Mech.* **54,** 599–611 (1972).
5. K. Bueller, K. R. Kirchartz and H. Oertel, Steady convection in a horizontal fluid layer, *Acta Mech.* **31,** 155–171 (1979).
6. P. Kolodner, P. R. Walden, A. Passner and C. M. Surko, Rayleigh–Bénard convection in an intermediate-aspect-ratio rectangular container, *J. Fluid Mech.* **163,** 195–226 (1986).
7. J. P. Gollub and S. V. Benson, Many routes to turbulent convection, *J. Fluid Mech.* **100,** 449–470 (1980).
8. J. Maurer and A. Libchaber, Rayleigh–Bénard experiment in liquid helium: frequency locking and the onset of turbulence, *J. Phys. Lett.* **40,** L419–423 (1979).
9. K. R. Kirchartz and H. Oertel, Three-dimensional thermal cellular convection in rectangular boxes, *J. Fluid Mech.* **192,** 249–286 (1988).
10. J. R. Leith, Successive transitions of steady states in moderate size containers of air heated from below and cooled above, *Bifurcation Phenomena in Thermal Processes and Convection*, HTD 94 (1987).
11. F. H. Busse and R. M. Clever, Instability of convection rolls in a fluid of moderate Prandtl number, *J. Fluid Mech.* **19,** 319–335 (1979).
12. W. V. R. Malkus, The heat transfer and spectrum of thermal turbulence, *Proc. R. Soc.* **A225,** 185–195 (1954).
13. C. D. Upson, P. M. Gresho, R. L. Sani, S. T. Chan and R. L. Lee, A thermal convection simulation in three dimensions by a modified finite element method. In *Numerical Properties and Methodologies in Heat Transfer* (Edited by T. M. Shih), pp. 245–259. Hemisphere, Washington, DC (1983).
14. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*. Hemisphere, Washington, DC (1980).
15. B. P. Leonard, A convectively stable, third-order accurate finite-difference method for steady two-dimensional flow and heat transfer. In *Numerical Properties and Methodologies in Heat Transfer* (Edited by T. M. Shih), pp. 211–226. Hemisphere, Washington, DC (1983).
16. J. P. Van Doormaal and G. D. Raithby, Enhancements of the simple method for predicting incompressible fluid flows, *Numer. Heat Transfer* **17,** 147–163 (1984).
17. R. Farhadieh and R. S. Tankin, Interferometric study of two-dimensional Bénard convection cells, *J. Fluid Mech.* **66,** 739–752 (1974).
18. D. Mukutmoni and K. T. Yang, Flow transitions in a three-dimensional rectangular enclosure heated from below, *ASME/JSME Thermal Engng Proc.*, Vol. 1, pp. 77–82 (1991).
19. D. Mukutmoni and K. T. Yang, Transition to oscillatory flow in Rayleigh–Bénard convection in a three-dimensional box, Paper 91-HT-10 of the National Heat Transfer Conf., Minneapolis (1991).
20. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*. Oxford University Press, Oxford (1961).
21. M. Giglio, S. Musazzi and U. Perini, Transition to chaotic behavior via a reproducible sequence of period-doubling bifurcations, *Phys. Rev. Lett.* **47,** 243–246 (1981).
22. A. Wolf, J. B. Swift, H. L. Swinney and J. A. Vastano, Determining Lyapunov exponents from a time series, *Physica* **16D,** 285–317 (1985).
23. G. Ahlers and R. P. Behringer, The Rayleigh–Bénard instability and the evolution of turbulence, *Suppl. Prog. Theor. Phys.* **64,** 186–201 (1978).
24. R. Krishnamurti, Some further studies on the transition to turbulent convection, *J. Fluid Mech.* **60,** 285–303 (1973).

SELECTION DE NOMBRE D'ONDE POUR LA CONVECTION DE RAYLEIGH–BENARD DANS UNE CAVITE A FAIBLE RAPPORT DE FORME

Résumé—On étudie numériquement la convection de Rayleigh–Bénard dans des cavités à faible rapport de forme et ayant des parois latérales isolées. On suppose un fluide be Boussinesq. On considère les bifurcations et la physique du champ d'écoulement pour trois rapports de forme 3,3 : 1,9 : 1, 3,5 : 2,1 : 1 et 4 : 2 : 1 et trois nombres de Prandtl 0,5, 0,71 et 2,5. On trouve que les rouleaux parallèles au plus long côté sont stables seulement pour un nombre de Rayleigh inférieur à 20 000 avec la géométrie 3,5 : 2,1 : 1 dans le cas de l'eau ($Pr = 2{,}5$). Avec la géométrie 3,3 : 1,9 : 1 et l'hélium liquide ($Pr = 0{,}5$), la transition de la configuration à trois cellules vers celle à deux cellules est accompagnée d'une grande dépendance au temps. Avec la géométrie 4 : 2 : 1 et l'air ($Pr = 0{,}71$), la transition entre quatre et trois rouleaux est un résultat de l'instabilité variqueuse oblique.

EINFLUSS DER WELLENZAHL AUF DIE RAYLEIGH-BENARD-KONVEKTION IN EINEM HOHLRAUM MIT KLEINEM SEITENVERHÄLTNIS

Zusammenfassung—Die vorliegende numerische Arbeit befaßt sich mit der Rayleigh–Benard-Konvektion in einem Hohlraum mit isolierten Seitenwänden und kleinem Seitenverhältnis. Es wird ein Boussinesq-Fluid zugrundegelegt. Für drei unterschiedliche Seitenverhältnisse (3,3 : 1,9 : 1 ; 3,5 : 2,1 : 1 ; 4 : 2 : 1) und drei unterschiedliche Prandtl-Zahlen (0,5 ; 0,71 ; 2,5) wurde die Strömungsaufteilung festgehalten und die Physik des Strömungsfeldes hinterleuchtet. Dabei hat sich herausgestellt, daß Strömungswalzen parallel zur Längsseite nur bei einer Rayleigh-Zahl von 20 000 für die Geometrie 3,5 : 2,1 : 1 mit Wasser ($Pr = 2,5$) als Versuchsstoff stabil sind. Bei der Geometrie 3,3 : 1,9 : 1 mit flüssigem Helium ($Pr = 0,5$) ist der Übergang von einer 3-zelligen zu einer 2-zelligen Struktur stark zeitabhängig. Bei der Geometrie 4 : 2 : 1 mit Luft ($Pr = 0,71$) ist der Übergang von einer 4-zelligen zu einer 3-zelligen Struktur das Ergebnis der "skewed-varicose"-Instabilität.

ВЫБОР ВОЛНОВОГО ЧИСЛА ДЛЯ КОНВЕКЦИИ РЭЛЕЯ-БЕНАРА В ПОЛОСТИ С МАЛЫМ ОТНОШЕНИЕМ СТОРОН

Аннотация—Численно исследуется конвекция Рэлея-Бенара в полостях с малым отношением сторон и изолированными боковыми стенками. Используется приплижение Буссинеска. Наблюдались разветвления течения и исследовались характеристики поля течения при трех различных отношениях сторон (3,3 : 1,9 : 1; 3,5 : 2,1 : 1 и 4 : 2 : 1) и трех различных числах Прандтля для жидкости (0,5; 0,71 и 2,5). Найдено, что валы, параллельные длинной стороне, устойчивы только при числе Рэлея ниже 20 000 для отношения сторон 3,5 : 2,1 : 1 с использованием воды ($Pr = 2,5$) в качестве рабочей жидкости. При отношении сторон 3,3 : 1,9 : 1 с использованием гелия ($Pr = 0,5$) для перехода от трехъячеистой структуры к двухъячеистой характерна сильная зависимость от времени. Обнаружено, что переход от четырех к трем валам при отношении сторон 4 : 2 : 1 с использованием воздуха ($Pr = 0,71$) обусловлен неустойчивостью в основании вала.